A Die Study of Victorian Shillings Dated 1865
Part 1 - Validating the Statistical Methods

Gary Oddie

Introduction

For some years the equations proposed by Warren Esty have been used to estimate the number of dies used to strike a particular issue or coinage[1,2]. The equations are used to give point estimates of the number of dies and the coverage and also 95% confidence limits on these numbers. Given a sample of a coinage the coverage is an estimate of how good or representative the sample is, and the number of dies estimated also includes those die for which no specimens have made it into the sample – the prediction. As with all statistical predictions (the weather, climate, card games etc) there is some uncertainty in the result and hence the 95% confidence limits, which might be considered error bars on the results.

On the whole, the results have always made sense when the samples are large and clearly random. The models give odd results for small and non-random samples (e.g. specific hoarding or culling events affecting survival into the sample). However, it is a model, and therefore the question has often been asked "do you believe the results?"

Motivation

The recent acquisition of a Victorian shilling dated 1865 with die number 102, reminded me of a fleeting interest many years ago in these curious issues. The piece will be described later in this note. However, more importantly, the shilling triggered a moment of clarity – the die numbers can be used to test the statistical methods because we have a good idea of what the answer is! As far as I am aware, this possibility has never been noticed and what follows later in this note has never been done before.

The die number experiment was carried out at the Royal Mint on the silver coinage between 1864 and 1879. For the shillings, every reverse die was given a unique number, in very small digits, entered by hand, just above the date as shown in figure 1.

Fig. 1. Details of the reverse of an 1865 shilling with die number 102, and close up.

This is all well known, as is the fact that there is no contemporary documentation giving the reasons for the inclusion of die numbers, or indeed any results from the project, if indeed the numbers were introduced to study die life.

The presence of die numbers has often just been noted in sales catalogues and many standard catalogues just note the highest die number known[3,4]. The early discoveries and listings were published in Coin Monthly in the 1970s[5-11]. The most recent listing has errors of inclusion and should be treated with caution[12]. In the decades following the 1970s, there has been a small group of highly dedicated collectors looking at this series.

The following table summarises the known die numbers for the shillings of 1865.

	1	2	3	4	5	6	7	8	9
10	**11**		**13**	**14**	**15**	**16**	**17**	**18**	**19**
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**
50	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**
60	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**
70	**71**	**72**		**74**	**75**	**76**	**77**	**78**	**79**
80	**81**		**83**	**84**	**85**	**86**	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**
100	**101**	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**
110	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**
120	**121**	**122**	**123**	**124**	**125**				
130									

Table 1. Known die numbers on shillings dated 1865. Black – normal occurrence, red extremely rare.

Whilst the majority of dies bearing die numbers are unique, there is a small exception. The 1878 shillings with die numbers 49, 51, 52 and 53 (50 not found) are known from multiple distinct dies (2 or 3). There is no such exception for 1865 and at present the maximum die number is 130 of which 122 have been found. The missing die numbers (12, 20, 73 and 82) may yet appear, but there is a possibility that the die failed when first used or if coins bearing these die numbers did make it into circulation they have been lost. Where blocks of die numbers are missing (126, 127, 128 and 129), there is a suspicion that this points to how the dies were ordered, engraved and used, possibly in small batches. These missing blocks often occur near the maximum and these dies may not have been put into service.

Statistical Method

Statistical models have been devised that allow an estimate of the number of dies used for a coinage, provided a reasonable sample can be examined, counted and individual dies identified[1,2].

Starting with the data from the die study

n = the sample size
d = the number of different dies in the sample
d_1 = the number of dies that struck just one coin in the sample, the singletons
d_i = the number of dies that struck i coins in the sample

We wish to estimate

D = the original number of dies used for the coinage
C = the coverage of the sample which is defined as

$$= \frac{\textit{The number of coins struck by dies represented in the sample}}{\textit{The number of coins struck by all of the dies}}$$

The coverage will tend towards unity as the sample becomes larger.

Table 2 summarises the equations for estimating the number of dies, the coverage and the 95% confidence limits of each.

	Point Estimate	95% Confidence Intervals
Number of Dies	$D_{est}=\left(\frac{d}{C_{est}}\right)\left(1+\frac{d_1}{2d}\right)$	Upper $D_+=D_{est}+\left(\frac{2D_{est}}{n}\right)^2+\left(\frac{2D_{est}}{n}\right)\sqrt{2D_{est}}$
		Lower $D_-=D_{est}+\left(\frac{2D_{est}}{n}\right)^2-\left(\frac{2D_{est}}{n}\right)\sqrt{2D_{est}}$
Coverage	$C_{est}=1-\frac{d_1}{n}$	Upper $C_+=C_{est}+\left(\frac{2}{n}\right)\sqrt{d_1+2d_2-\frac{d_1^2}{n}}$
		Lower $C_-=C_{est}-\left(\frac{2}{n}\right)\sqrt{d_1+2d_2-\frac{d_1^2}{n}}$

Table 2. Equations for estimating the number of dies, the coverage and the 95% confidence limits of each.

The Sample

In the usual way the online sources were scoured for shillings dated 1865. Several evenings were casually spent working through eBay, the archives of the auction houses (Noonans and London Coin Auctions) and dealers' stocks, gathering pictures of shillings dated 1865 with clear images of die numbers. A simple file-naming method allowed duplicate entries to be spotted when a piece has reappeared for sale, sometimes weeks or even years apart.

As a side note, it is very clear that if you want to collect die numbers, you really cannot be choosy about the grades that you collect. The pieces from the larger auction houses are typically EF, or even when in groups, VF, in order to make the minimum lot value. The majority of the sample came from eBay where grades ranged from poor to EF, within these the majority are less than Fine.

Ultimately images of 184 different shillings dated 1865 were captured and this is the data upon which the statistical methods will be tested. As the images are gathered, the files are date-stamped and the filename includes the observed die number – the unique identifier of that die. The exact die numbers do not matter, all that is required is a way to identify different dies and die duplicates. Just like a real (and quite costly) collection, the "virtual collection" of 1865 shillings can thus be sorted by date of finding and also the unique identifying feature (the die number).

As expected, the virtual collection begins with the accumulation of singletons and the first die duplicate appears when there are 10 coins in the sample. The table below shows the evolution of the sample as the numbers increase.

n	10	25	50	75	100	125	150	184
d	9	20	35	47	56	64	70	78
d_1	8	15	24	28	29	28	28	30
d_2	1	5	8	14	15	22	23	20
d_3			2	3	9	9	11	16
d_4			1		1	3	2	4
d_5				2	2		4	3
d_6						1	1	3
d_7								1
d_8						1		
d_9							1	
d_{10}								1

Table 3. Progression of the virtual collection of 1865 shillings with increasing sample size *n*.

Thus with 184 pieces in the collection, one of the dies was known from 10 different specimens. Table 4 adds other parameters from the sample such as lowest/highest die numbers seen in the sample.

n	10	25	50	75	100	125	150	184
d	9	20	35	47	56	64	70	78
Lowest Die Number	6	6	5	1	1	1	1	1
Highest Die Number	115	123	125	125	125	125	125	125

Table 4. Progression of the number of different dies and range of observed Die Numbers.

The chart below includes the data from Table 4 and superimposes lines to show that we know that the total number of dies will be somewhere between the 122 observed and the 130 that might be expected to exist (derived from Table 1).

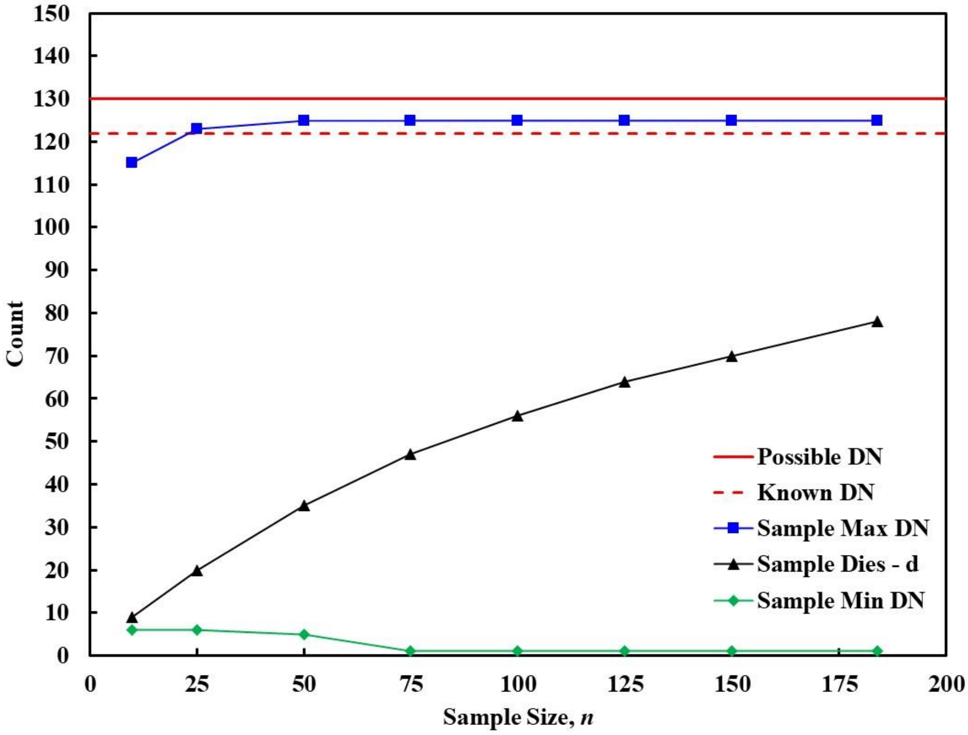

Fig. 2. Analysing the contents of the virtual collection as its numbers grew.

Thus by the time the collection contained 75 images, though there were only 47 different dies, the range of die numbers was (DN min to DN max) was 1 to 125. Even when the collection comprised 184 images, just 78 different dies had been found and the line indicating the number of dies found is definitely showing signs of levelling off. This again confirms the dedication required (and a sample size certainly in the high hundreds and possibly thousands) to produce the data in Table 1. Note that in the sample, the maximum die number seen was 125. A specimen of the highest known (130) had not yet been seen.

Although the virtual collection is far from complete, the equations in Table 2 can be used to estimate the likely number of dies used and the quality of the sample used for the analysis.

Statistical Analysis

The data in Table 3 is entered into the equations in Table 2 for each stage of the virtual collection. Table 5 gives all of the results, with the coverage given to two decimal places and the die estimates rounded to the nearest integer.

n	10	25	50	75	100	125	150	184
C_{est}	0.20	0.40	0.52	0.63	0.71	0.78	0.81	0.84
C_+	0.58	0.72	0.73	0.81	0.85	0.91	0.92	0.92
C_-	-0.18	0.08	0.31	0.45	0.57	0.65	0.70	0.75
D_{est}	65	69	90	97	99	101	103	111
D_+	382	163	152	140	131	126	125	131
D_-	86	35	55	68	75	80	85	95

Table 5. Results of the statistical analysis of the virtual collection as the collection grew.

The data is much easier to interpret on charts. Firstly, how good is the sample, the coverage? Figure 3 plots the coverage of the sample and its 95% confidence limits as a function of the size of the sample, *n*.

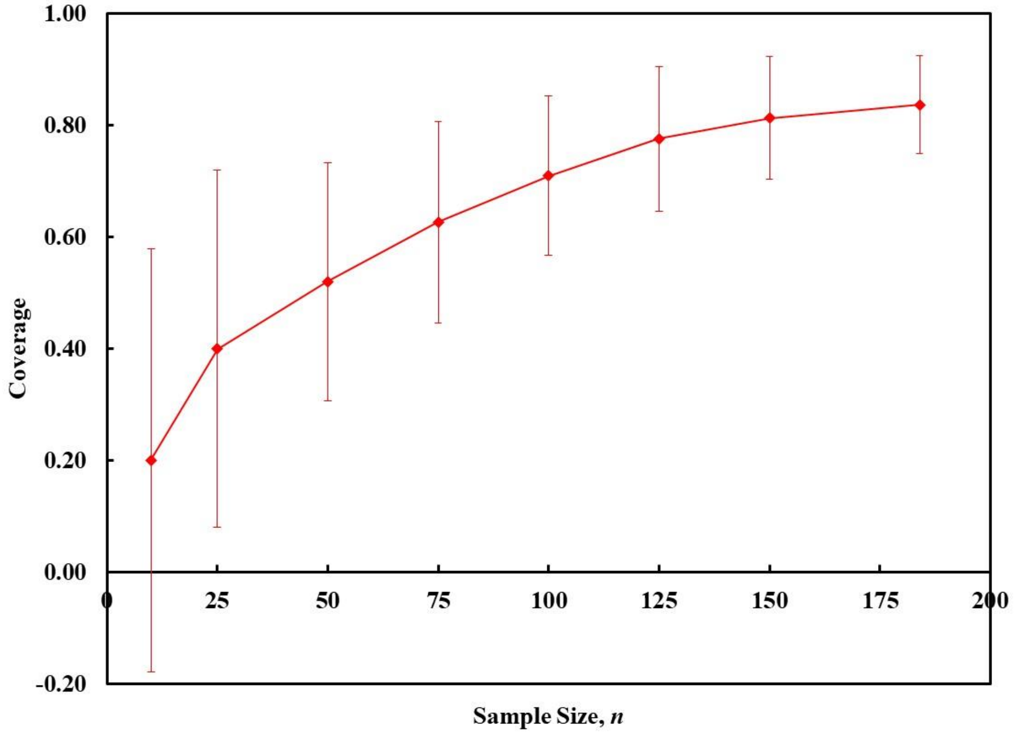

Fig. 3. Estimates of the quality of the sample, the coverage, and 95% confidence limits, as the sample grew.

The obvious problem is the results when there were just 10 coins in the sample and just one die duplicate. A coverage of 0.20 is poor and a lower 95% confidence limit that is negative is just not realistic and a warning to be very careful when applying statistical methods to small samples.

As the size of the sample increases, there is a steady increase in the coverage and a narrowing of the 95% confidence limits (the error bars). With a sample of 184 coins the coverage is 0.84, but the trend is levelling off, again confirming that a very large sample is needed to approach C_{est}=1.00.

Figure 4 plots the predicted/estimated number of dies and the 95% confidence limits as the sample grew.

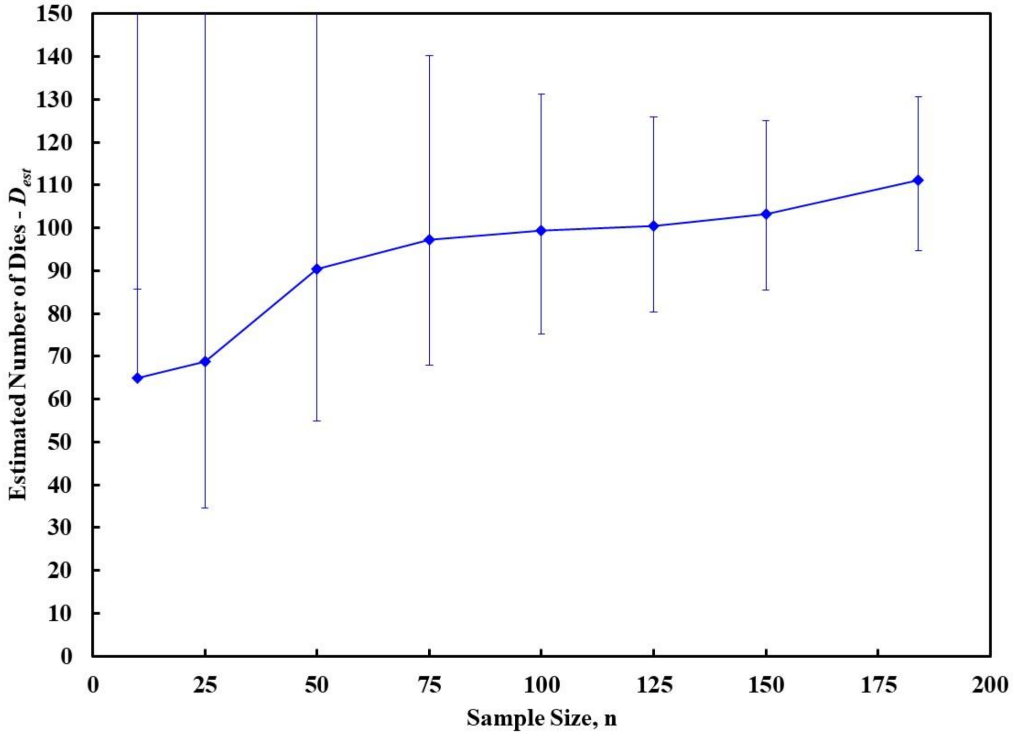

Fig. 4. Estimated number of 1865 dies and 95% confidence limits as a function of the sample size.

Once again, when the sample is small, the predictions/estimates have spurious confidence limits, but once the sample becomes larger (better coverage) the estimated number of dies climbs gently and the 95% confidence limits become narrower.

To check how accurate this predicted number of dies is, the following plot includes lines for the already observed number of dies (122), the possible maximum number of dies (130) and number of dies in the simple (Table 3, Figure 2).

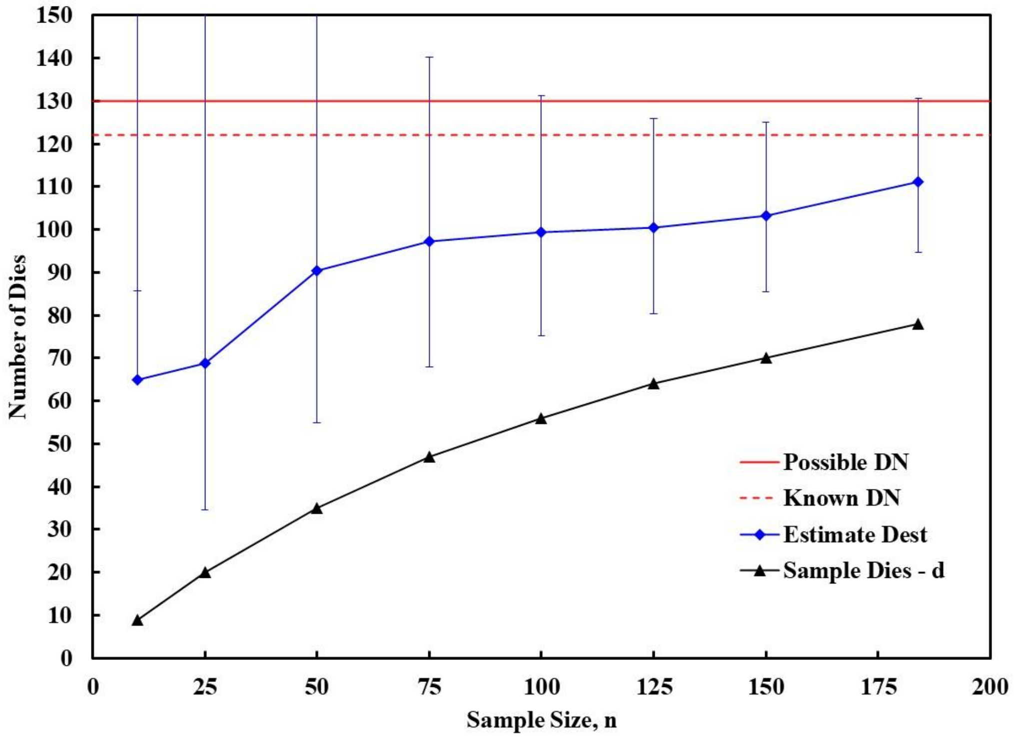

Fig. 5. Comparing the Estimated number of dies with the known and possible number of dies.

Thus, from a sample of 184 coins which included 78 different dies, the statistical equations are estimating that the number of dies is in the range 95 to 131 (95% confidence limits) with a point estimate D_{est} of 111 dies. That the number of known dies is 122, and may be as high as 130, confirms that the statistical methods do give quite reasonable estimates from relatively small samples and a coverage of 0.84.

Significantly Increasing the Sample Size

All of the data presented so far has been based on a quickly gathered snap-shot of the 1865 shillings that could be found online – 184 pieces. The various curves can be seen moving towards the expected values: sample dies (d) and estimated dies (d_{est}) towards the known and possible values in the range 122-130 and the coverage (C_{est}) slowly approaching 1.00.

The next step is to significantly increase the sample size. This can be achieved by collecting data of further specimens of 1865 shillings for several decades. It is important that the data is collected without the bias typical of many collectors and museums, for example acquiring just one of each variety, and possibly upgrading and discarding as opportunities arise. Such collections are systematically biased towards the rarities and away from the commoner pieces. In the limit many collections would be comprised only of singletons.

A truly random collection should incorporate no biases and as such will include many singletons, die duplicates, triplicates etc. Fortunately, such a collection of 1865 die numbered shillings was put together by David Morley prior to 2014. The data also includes details provided by other collectors including Ron Stafford in the decades following his original publications on the subject[10,11]. The data has been kindly provided for this study and comes in three parts as follows:

(1) Pieces dated 1865 in the David Morley collection (341). This has a very high confidence in the readings of the die numbers.
(2) Data of other pieces dated 1865 (98) from other very reliable sources.
(3) Data of other pieces dated 1865 (325) from other sources. Whilst most of these will be good readings, there are expected to be a few errors that would be picked up by a specialist examination and comparison with other pieces claiming the same die numbers. The opportunity to do this has passed and so the data has to be taken at face value.

There is a very good chance that some of the pieces in the above three independent groups have made it into the sample of 184 pieces presented in the first part of the analysis and thus the additional data must be treated separately to avoid double counting.

The additional data is used to form two new groups. The first group includes the most reliable die identifications (i.e. 341 + 98 = 439 pieces) and the second combines all of the additional data (i.e. 341 + 98 + 325 = 764 pieces). The table below gives the results of the usual statistical analysis of the two new groups.

n	439	764
C_{est}	0.95	0.98
C_{+}	0.99	1.00
C_{-}	0.92	0.96
D_{est}	130	131
D_{+}	140	137
D_{-}	121	126

Table 6. Results of the statistical analysis of the additional data for 1865 shillings.

The two new data sets have increased the 1865 shilling sample size from 184 to 439 and finally 764 different pieces. This has resulted in the coverage becoming very close to unity and the estimated number of dies becoming very close to the maximum number of expected dies (130).

The trends become clear when the results of the analysis of the additional data are appended to the previous charts of Coverage and Numbers of dies.

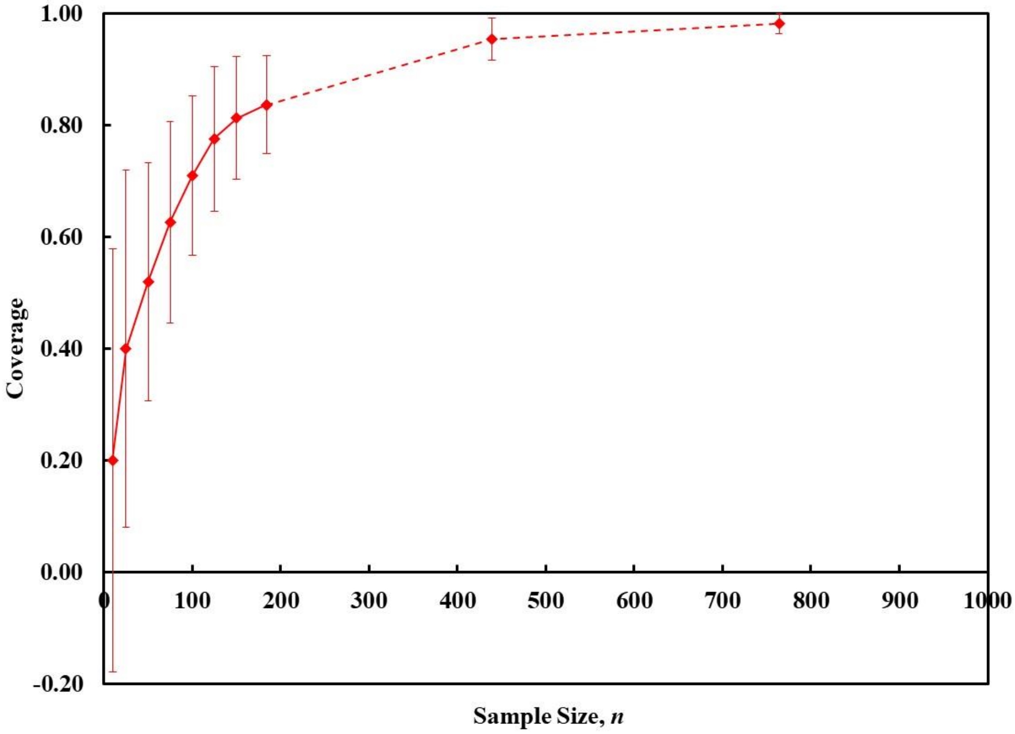

Fig. 6. Estimates of the quality of the sample, the coverage, and 95% confidence limits, for the original and additional data sets.

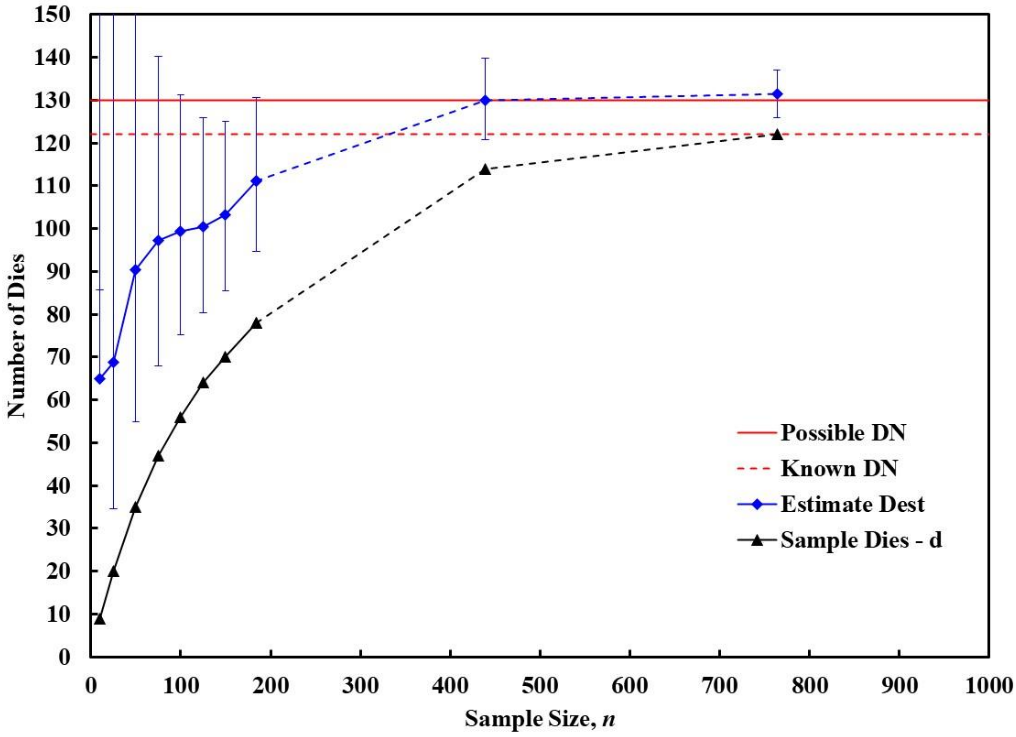

Fig. 7. Comparing the Estimated number of dies with the known and possible number of dies for the original and additional data sets.

The above two charts confirm that the statistical methods do give the correct answer when the sample is large, and the coverage approaches unity.

A Rare Die Number

The motivation behind this note followed the acquisition of the piece shown above in Figure 1 and being told it was an extremely rare die number with just one other noted with certainty. The image was cropped so as not to distract, and the full coin is shown below, and its state probably accounts for the rarity of coins from this die.

Fig. 6. Obverse and reverse of 1865 die number 102 shilling.

The reverse die has clearly undergone a significant failure with extensive die flaws and die cuds. The whole flan is distorted in the area of the die flaw. Being the upper die, the fragments of metal have fallen into the obverse die below and have become a hardened steel grinding powder. This has created scratches and indents in the die field and design resulting in raised lines and incuse marks on most of the surface of the struck coin.

The rarity of this die number suggests the failure happened much earlier than the normal die life expectancy. The die would have been discarded and damaged coins put back in the melting pot. This coin escaped into circulation.

Conclusions

The acquisition of a rare and mis-struck shilling of 1865, bearing the die number 102, has prompted this short study into validating the statistical methods used to estimate the number of dies used to strike a coinage.

Conveniently, the presence of die numbers allows dies to be easily identified and also the original numbers of dies to be estimated. As the original Royal Mint records have no relevant data, collectors over the past half century have created lists of known die numbers.

In the first part of the note, a virtual collection of 184 images of shillings dated 1865 has been created to test the statistical method. When the sample is small, as expected, the analysis gives spurious results. However once the coverage exceeds about 0.5, the estimated number of dies starts to increase and converge on the expected value.

With this virtual collection the maximum coverage is 0.84, the statistical equations estimate that the number of dies is in the range 95 to 131 (95% confidence limits) with a point estimate D_{est} of 111 dies. That the number of known dies is 122, and may be as high as 130, confirms that the statistical methods do give quite reasonable estimates from relatively small samples and a coverage of 0.84. This result is a reasonable, but not a conclusive test of the statistical methods.

In the second part of the note, data from a much larger sample of shillings dated 1865 has been analysed (764 pieces). With this collection the maximum coverage is 0.98, and the statistical equations estimate that the number of dies is in the range 126 to 137 (95% confidence limits) with a point estimate D_{est} of 131 dies. That the number dies known today is 122, but may be as high as 130, confirms that the statistical methods can give very good estimates provided the sample size is large enough and the coverage approaches unity.

In summary, the statistical methods have been validated and the first practical use has been found for the die number experiment.

References and Acknowledgements

1. W.W. Esty. Estimation of the size of a coinage: A survey and comparison of methods. *Numismatic Chronicle.* Vol 146 (1986) pp185-215.
2. W.W. Esty. How to estimate the original number of dies and the coverage of a sample. *Numismatic Chronicle*. Vol. 166 (2006), pp. 359-364.
3. H.A. Seaby and P.A. Rayner. *English Silver Coinage from 1649*. 4th Edition, Seaby, 1974.
4. P.A. Rayner. *English Silver Coinage since 1649*. 5th Edition, Seaby, 1992.
5. M. Mapleton. *Die Numbers on Victorian Coins – Part 1*. Coin Monthly, February 1971 pp27-29.
6. M. Mapleton. - *Part 2 - Shillings*. Coin Monthly, March 1971 pp49-54.
7. M. Mapleton. - *Part 3 - Florins*. Coin Monthly, April 1971 pp29-33.
8. M. Mapleton. - *Part 4 - Sixpences*. Coin Monthly, May 1971 pp51-53.
9. M. Mapleton. - *Part 5 - Conclusions*. Coin Monthly, June 1971 p33.
10. R. Stafford. *A Survey of Die Numbers – Part 1*. Coin Monthly, March 1977 pp29-36.
11. R. Stafford. *A Survey of Die Numbers – Part 2*. Coin Monthly, April 1977 pp45-52.
12. M. Bull. *English Silver Coinage since 1649*. 6th Edition, Spink, 2015.

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